

Next-to-leading order QCD corrections to the top quark decay via the Flavor-Changing Neutral-Current operators with mixing effects

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Abstract

In this paper detailed calculations of the complete $\mathcal{O}(\alpha_s)$ corrections to top quark decay widths $\Gamma(t \rightarrow q + V)$ are presented ($V = g, \gamma, Z$). Besides describing in detail the calculations in our previous paper (arXiv:0810.3889), we also include the mixing effects of the Flavor-Changing Neutral-Current (FCNC) operators for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, which were not considered in our previous paper. The results for $t \rightarrow q + g$ are the same as in our previous paper. But the mixing effects can either be large or small, and increase or decrease the branching ratios for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, depending on the values of the anomalous couplings ($\kappa_{tq}^{g,\gamma,Z}/\Lambda$, $f_{tq}^{g,\gamma,Z}$ and $h_{tq}^{g,\gamma,Z}$).

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I. INTRODUCTION

Physics beyond the standard model (SM) can manifest itself by altering the expected rates of Flavor-Changing Neutral-Current (FCNC) interactions. Thus, testing SM and probing new physics through top quark FCNC decay is interesting. The top quark FCNC processes $t \rightarrow q + V$ ($V = g, \gamma, Z$) have tiny branching ratios in the SM and are probably unmeasurable at the CERN Large Hadron Collider (LHC) and future colliders. Therefore, any positive signal of these rare decay events would imply new physics beyond the SM. As the LHC will produce abundant top quark events (about 10^8 per year), even in the initial low luminosity run ($\sim 10\text{fb}^{-1}/\text{year}$) 8×10^6 top-quark pairs and 3×10^6 single top quarks will be produced yearly. Thus one may anticipate to discover the first hint of new physics by observing anomalous couplings in the top-quark sector.

Recently, from their measurements of the total cross sections, both D0 [1] and CDF [2] collaborations at the Fermilab Tevatron have searched for nonstandard-model single top quark production and set upper limits on the anomalous FCNC couplings κ_{tc}^g/Λ and κ_{tu}^g/Λ , where the leading order (LO) cross sections have been scaled to next-to-leading order (NLO) [3] (or resummation [4]) predictions. At the LHC, ATLAS collaboration has presented its sensitivity by studying FCNC top decays [5]. These results show that top quark FCNC couplings will provide a good probe to new physics beyond the SM. Although there are many discussions in the literatures on top quark production and rare decay processes involving model-independent top quark FCNC couplings [6–18], most of them were based on LO calculations. However, especially for $t \rightarrow q + g$, due to the large uncertainties from the renormalization scale dependence in its LO prediction through the strong coupling constant α_s , it is necessary to improve the theoretical prediction to NLO in order to match the expected experimental accuracy at the LHC. Because of the importance of NLO corrections for the experiments [2], we calculated the NLO QCD corrections to the partial decay widths and decay branching ratios of top quark FCNC processes $t \rightarrow q + V$ ($V = g, \gamma, Z$) more than one year ago [19].

In this paper, we describe in detail the calculations in Ref. [19] and consider the FCNC operator mixing effects which were ignored in Ref. [19]. This paper is organized as follows. In section II, we show the relevant dimension-five operators and the corresponding LO results in the top sector. Section III is devoted to the detail of the calculation presented in Ref. [19].

Section IV deals with the evolution of anomalous couplings κ_{tq}^g . The analytic results for the mixing effects can be found in section V, while the numerical results are presented in section VI.

II. LEADING ORDER RESULTS

New physics effects involved in top quark FCNC processes can be incorporated in a model-independent way, into an effective Lagrangian which includes the dimension-five operators as listed below [8]

$$\begin{aligned} \mathcal{L}^{\text{eff}} = & -\frac{e}{\sin 2\theta_W} \sum_{q=u,c} \frac{\kappa_{tq}^Z}{\Lambda} \bar{q} \sigma^{\mu\nu} (f_{tq}^Z + i h_{tq}^Z \gamma_5) t Z_{\mu\nu} - e \sum_{q=u,c} \frac{\kappa_{tq}^\gamma}{\Lambda} \bar{q} \sigma^{\mu\nu} (f_{tq}^\gamma + i h_{tq}^\gamma \gamma_5) t A_{\mu\nu} \\ & - g_s \sum_{q=u,c} \frac{\kappa_{tq}^g}{\Lambda} \bar{q} \sigma^{\mu\nu} T^a (f_{tq}^g + i h_{tq}^g \gamma_5) t G_{\mu\nu}^a + \text{H.c.} \end{aligned} \quad (1)$$

where Λ is the new physics scale, θ_W is the weak-mixing angle, and T^a are the conventional Gell-Mann matrices. The coefficients κ_{tq}^Z , κ_{tq}^γ and κ_{tq}^g are normalized to be real and positive, while $f_{tq}^V, h_{tq}^V (V = Z, \gamma, g)$ are complex numbers satisfying $|f|^2 + |h|^2 = 1$.

From the effective Lagrangian above, we obtain the following LO partial decay width of the FCNC top decay in the $D = 4 - 2\epsilon$ dimension,

$$\begin{aligned} \Gamma_0^\epsilon(t \rightarrow q + g) &= \frac{8\alpha_s m_t^3}{3} \left(\frac{\kappa_{tq}^g}{\Lambda} \right)^2 C_\epsilon, \\ \Gamma_0^\epsilon(t \rightarrow q + \gamma) &= 2\alpha m_t^3 \left(\frac{\kappa_{tq}^\gamma}{\Lambda} \right)^2 C_\epsilon, \\ \Gamma_0^\epsilon(t \rightarrow q + Z) &= \frac{\alpha m_t^3 \beta_Z^{4-4\epsilon}}{\sin^2 2\theta_W} \left(\frac{\kappa_{tq}^Z}{\Lambda} \right)^2 (3 - \beta_Z^2 - 2\epsilon) \frac{C_\epsilon}{1 - \epsilon}, \end{aligned} \quad (2)$$

where the masses of light quarks q ($q = u, c$) have been neglected, and $\beta_Z = \sqrt{1 - M_Z^2/m_t^2}$, $C_\epsilon = \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} (\frac{4\pi\mu^2}{m_t^2})^\epsilon$. Here, we also define $\Gamma_0(t \rightarrow q + V) = \Gamma_0^\epsilon(t \rightarrow q + V)|_{\epsilon \rightarrow 0}$, which are consistent with the results of Refs. [5, 8].

III. NEXT-TO-LEADING ORDER RESULTS

Below, we present our calculation in detail for the inclusive decay width of the top quark, up to the NLO with the LO partonic process denoted as $t \rightarrow q + g$. The results of $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$ are similar to the $t \rightarrow q + g$, and will be presented all together.

At the NLO, we need to include both one-loop virtual gluon corrections (Fig. 2) and real gluon emission contribution (Fig. 3). We use dimensional regularization to regulate both ultraviolet (UV) and infrared (IR) (soft and collinear) divergences, with spatial-time dimension $D = 4 - 2\epsilon$. The UV divergences cancel after summing up the contributions from the one-loop virtual diagrams and counterterms according to the same convention used in Ref. [3]. The soft divergences cancel after adding up the virtual and real radiative corrections. To cancel collinear singularities, we need to also include contribution induced from gluon splitting to light quark pairs at the same order in the QCD coupling.

The virtual correction of $t \rightarrow q + g$ contains UV and IR divergences, which has the same form as in Ref. [3] and can be written as (the imaginary part is neglected):

$$\begin{aligned} \mathcal{M}_{\text{virt}} = & \frac{\alpha_s}{12\pi} D_\epsilon \left(-\frac{13}{\epsilon_{\text{IR}}^2} - \frac{17}{\epsilon_{\text{IR}}} + \frac{11}{\epsilon_{\text{UV}}} + \frac{17}{3}\pi^2 - 15 \right) \mathcal{M}_0 \\ & + \left(\frac{1}{2}\delta Z_2^{(g)} + \frac{1}{2}\delta Z_2^{(q)} + \frac{1}{2}\delta Z_2^{(t)} + \delta Z_{g_s} + \delta Z_{\kappa_{tq}^g/\Lambda} \right) \mathcal{M}_0, \end{aligned} \quad (3)$$

where $D_\epsilon = \Gamma(1 + \epsilon)[(4\pi\mu^2)/m_t^2]^\epsilon$, and the UV divergences are renormalized by introducing counterterms for the wave function of the external fields ($\delta Z_2^{(g)}, \delta Z_2^{(q)}, \delta Z_2^{(t)}$) and the coupling constants ($\delta Z_{g_s}, \delta Z_{\kappa_{tq}^g/\Lambda}$). We define these counterterms according to the same convention as in Ref.[3]:

$$\begin{aligned} \delta Z_2^{(g)} &= -\frac{\alpha_s}{2\pi} D_\epsilon \left(\frac{N_f}{3} - \frac{5}{2} \right) \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) - \frac{\alpha_s}{6\pi} D_\epsilon \frac{1}{\epsilon_{\text{UV}}}, \\ \delta Z_2^{(q)} &= -\frac{\alpha_s}{3\pi} D_\epsilon \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right), \\ \delta Z_2^{(t)} &= -\frac{\alpha_s}{3\pi} D_\epsilon \left(\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} + 4 \right), \\ \delta Z_{g_s} &= \frac{\alpha_s}{4\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \left(\frac{N_f}{3} - \frac{11}{2} \right) \frac{1}{\epsilon_{\text{UV}}} + \frac{\alpha_s}{12\pi} D_\epsilon \frac{1}{\epsilon_{\text{UV}}}, \\ \delta Z_{\kappa_{tq}^g/\Lambda} &= \frac{\alpha_s}{6\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \frac{1}{\epsilon_{\text{UV}}}, \\ \delta Z_{\kappa_{tq}^\gamma/\Lambda} &= \delta Z_{\kappa_{tq}^Z/\Lambda} = \frac{\alpha_s}{3\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \frac{1}{\epsilon_{\text{UV}}}, \end{aligned} \quad (4)$$

where $N_f = 5$ is the number of the active quark flavors. Here we also present the necessary definition $\delta Z_{\kappa_{tq}^\gamma/\Lambda}$ and $\delta Z_{\kappa_{tq}^Z/\Lambda}$ used in the calculations of the decay modes $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$. Combine these together and integrate through phase space, we get the virtual

contributions without UV divergences:

$$\begin{aligned} \Gamma_{\text{virt}}^g = & \frac{\alpha_s}{6\pi} \Gamma_0^\epsilon(t \rightarrow q + g) \left\{ -\frac{13}{\epsilon_{\text{IR}}^2} + \frac{1}{\epsilon_{\text{IR}}} \left[-13 \ln \frac{4\pi\mu^2}{m_t^2} + 13\gamma_E + N_f - \frac{53}{2} \right] \right. \\ & \left. + \left[-\frac{13}{2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right)^2 - 12 \ln \frac{\mu^2}{m_t^2} + \left(N_f - \frac{53}{2} \right) (\ln 4\pi - \gamma_E) + \frac{55\pi^2}{12} - 23 \right] \right\}. \end{aligned} \quad (5)$$

All the UV divergences have canceled in Γ_{virt}^g , as they must, but the infrared divergent piece is still present. We also show the corresponding results for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$ below,

$$\begin{aligned} \Gamma_{\text{virt}}^\gamma = & \frac{\alpha_s}{3\pi} \Gamma_0^\epsilon(t \rightarrow q + \gamma) \left\{ -\frac{2}{\epsilon_{\text{IR}}^2} + \frac{1}{\epsilon_{\text{IR}}} \left[-2 \ln \frac{4\pi\mu^2}{m_t^2} + 2\gamma_E - 5 \right] - 2 \ln \frac{\mu^2}{m_t^2} \right. \\ & \left. - \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right)^2 + 5 \left(\gamma_E - \ln \frac{4\pi\mu^2}{m_t^2} \right) - \frac{\pi^2}{6} - 12 \right\}, \\ \Gamma_{\text{virt}}^Z = & \frac{\alpha_s}{3\pi} \Gamma_0^\epsilon(t \rightarrow q + Z) \left\{ -\frac{2}{\epsilon_{\text{IR}}^2} + \frac{1}{\epsilon_{\text{IR}}} \left[8 \ln \beta_Z - 2 \ln \frac{4\pi\mu^2}{m_t^2} + 2\gamma_E - 5 \right] - 2 \ln \frac{\mu^2}{m_t^2} \right. \\ & + 8 \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E + \frac{3 - 2\beta_Z^2}{3 - \beta_Z^2} \right) \ln \beta_Z - \ln \frac{4\pi\mu^2}{m_t^2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - 2\gamma_E + 5 \right) - 8 \ln^2 \beta_Z \\ & \left. + 4\text{Li}_2 \left(-\frac{1 - \beta_Z^2}{\beta_Z^2} \right) - \gamma_E^2 + 5\gamma_E - \frac{\pi^2}{6} - 12 \right\}. \end{aligned} \quad (6)$$

The contribution from real gluon emission ($t \rightarrow q + g + g$) is denoted as $\Gamma_{\text{real}}(t \rightarrow q + g + g)$. In order to cancel all the collinear divergences in the sum of virtual and real radiative corrections, we also need to include the contributions from gluon splitting into a pair of quark and antiquark in the collinear region, which is denoted as $\Gamma_{\text{real}}(t \rightarrow q + q' + \bar{q}')$. Note that there are two configurations of final state when the flavor of the light quark coming from gluon splitting is the same as the light quark directly from the FCNC coupling, and only one configuration when they are different. The contributions of real gluon emission ($t \rightarrow q + g + g$) and gluon splitting ($t \rightarrow q + q' + \bar{q}'$) are, respectively,

$$\begin{aligned} \Gamma_{\text{real}}(t \rightarrow q + g + g) = & \frac{\alpha_s}{6\pi} \Gamma_0^\epsilon(t \rightarrow q + g) \left[\frac{13}{\epsilon_{\text{IR}}^2} + \frac{1}{\epsilon_{\text{IR}}} \left(13 \ln \frac{4\pi\mu^2}{m_t^2} - 13\gamma_E + \frac{53}{2} \right) \right. \\ & \left. + \frac{13}{2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right)^2 + \frac{53}{2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right) - \frac{31}{4}\pi^2 + \frac{171}{2} \right], \end{aligned} \quad (7)$$

and,

$$\Gamma_{\text{real}}(t \rightarrow q + q' + \bar{q}') = \frac{\alpha_s}{6\pi} \Gamma_0^\epsilon(t \rightarrow q + g) \left[-\frac{1}{\epsilon_{\text{IR}}} N_f + N_f \left(\gamma_E - \ln \frac{4\pi\mu^2}{m_t^2} - 3 \right) - \frac{1}{12} \right]. \quad (8)$$

After adding them together, the total real contributions can be written as

$$\begin{aligned}
\Gamma_{\text{real}}^g &= \Gamma_{\text{real}}(t \rightarrow q + g + g) + \Gamma_{\text{real}}(t \rightarrow q + q' + \bar{q}') \\
&= \frac{\alpha_s}{6\pi} \Gamma_0^\epsilon(t \rightarrow q + g) \left\{ \frac{13}{\epsilon_{\text{IR}}^2} - \frac{1}{\epsilon_{\text{IR}}} \left[-13 \ln \frac{4\pi\mu^2}{m_t^2} + 13\gamma_E + N_f - \frac{53}{2} \right] \right. \\
&\quad + \left[\frac{13}{2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right)^2 + \frac{53}{2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right) \right. \\
&\quad \left. \left. - N_f \left(\ln \frac{4\pi\mu^2}{m_t^2} + 3 - \gamma_E \right) - \frac{31\pi^2}{4} + \frac{1025}{12} \right] \right\}. \tag{9}
\end{aligned}$$

The corresponding real contributions of $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$ are also shown below

$$\begin{aligned}
\Gamma_{\text{real}}^\gamma &= \frac{\alpha_s}{3\pi} \Gamma_0^\epsilon(t \rightarrow q + \gamma) \left\{ \frac{2}{\epsilon_{\text{IR}}^2} + \frac{1}{\epsilon_{\text{IR}}} \left[2 \ln \frac{4\pi\mu^2}{m_t^2} - 2\gamma_E + 5 \right] \right. \\
&\quad \left. + \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right)^2 + 5 \left(\ln \frac{4\pi\mu^2}{m_t^2} - \gamma_E \right) - \frac{7\pi^2}{6} + \frac{52}{3} \right\}, \\
\Gamma_{\text{real}}^Z &= \frac{\alpha_s}{3\pi} \Gamma_0^\epsilon(t \rightarrow q + Z) \left\{ \frac{2}{\epsilon_{\text{IR}}^2} + \frac{1}{\epsilon_{\text{IR}}} \left[-8 \ln \beta_Z + 2 \ln \frac{4\pi\mu^2}{m_t^2} - 2\gamma_E + 5 \right] \right. \\
&\quad + \left[4(4 \ln \beta_Z + 2\gamma_E - 5) \ln \beta_Z + \ln \frac{4\pi\mu^2}{m_t^2} \left(\ln \frac{4\pi\mu^2}{m_t^2} - 8 \ln \beta_Z - 2\gamma_E + 5 \right) + 4\text{Li}_2(\beta^2) \right. \\
&\quad \left. \left. + \frac{4(1 - \beta_Z^2)(1 - 4\beta_Z^2 + \beta_Z^4)}{\beta_Z^4(3 - \beta_Z^2)} \ln(1 - \beta_Z^2) + \frac{12 + 135\beta_Z^2 - 43\beta_Z^4}{3\beta_Z^2(3 - \beta_Z^2)} + \gamma_E^2 - 5\gamma_E - \frac{11\pi^2}{6} \right] \right\}. \tag{10}
\end{aligned}$$

Combine the real and virtual contributions, we obtain the full NLO corrections for $t \rightarrow q + g$, $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$ as

$$\begin{aligned}
\Gamma_{\text{NLO}}(t \rightarrow q + g) &= \Gamma_{\text{virt}}^g + \Gamma_{\text{real}}^g \\
&= \frac{\alpha_s}{72\pi} \Gamma_0^\epsilon(t \rightarrow q + g) \left[174 \ln \left(\frac{\mu^2}{m_t^2} \right) - 12N_f \ln \left(\frac{\mu^2}{m_t^2} \right) - 36N_f - 38\pi^2 + 749 \right], \\
\Gamma_{\text{NLO}}(t \rightarrow q + \gamma) &= \Gamma_{\text{virt}}^\gamma + \Gamma_{\text{real}}^\gamma \\
&= \frac{\alpha_s}{9\pi} \Gamma_0^\epsilon(t \rightarrow q + \gamma) \left[-6 \ln \left(\frac{\mu^2}{m_t^2} \right) - 4\pi^2 + 16 \right], \\
\Gamma_{\text{NLO}}(t \rightarrow q + Z) &= \Gamma_{\text{virt}}^Z + \Gamma_{\text{real}}^Z \\
&= \frac{\alpha_s}{3\pi} \Gamma_0^\epsilon(t \rightarrow q + Z) \left[-2 \ln \left(\frac{\mu^2}{m_t^2} \right) + \frac{4(1 - \beta_Z^2)(1 - 4\beta_Z^2 + \beta_Z^4)}{\beta_Z^4(3 - \beta_Z^2)} \ln(1 - \beta_Z^2) \right. \\
&\quad + 4 \left(2 \ln \beta_Z - \frac{9 - \beta_Z^2}{3 - \beta_Z^2} \right) \ln \beta_Z + 4\text{Li}_2(\beta_Z^2) + 4\text{Li}_2 \left(-\frac{1 - \beta_Z^2}{\beta_Z^2} \right) \\
&\quad \left. + \frac{12 + 135\beta_Z^2 - 43\beta_Z^4}{3\beta_Z^2(3 - \beta_Z^2)} - 2\pi^2 - 12 \right]. \tag{11}
\end{aligned}$$

Note that the last expression in Eq. (11) differs from the Eq. (9) in Ref. [19], and the numerical difference is about 2%, as shown below. But the conclusion on branching ratio

for $t \rightarrow q + Z$ in Ref. [19] is not changed, i.e., the NLO correction is minuscule in branching ratio for $t \rightarrow q + Z$. Thus, up to the NLO, the partial decay width of the three FCNC decays can be obtained

$$\Gamma(t \rightarrow q + V) = \Gamma_0(t \rightarrow q + V) + \Gamma_{\text{NLO}}(t \rightarrow q + V). \quad (12)$$

In order to study the effects of NLO corrections to the decay branching ratios, we define the following branching ratios for later numerical analysis:

$$\begin{aligned} \text{BR}_{\text{LO}}(t \rightarrow q + V) &= \frac{\Gamma_0(t \rightarrow q + V)}{\Gamma_0(t \rightarrow W + b)}, \\ \text{BR}_{\text{NLO}}(t \rightarrow q + V) &= \frac{\Gamma(t \rightarrow q + V)}{\Gamma(t \rightarrow W + b)}. \end{aligned} \quad (13)$$

The decay width for the dominant top quark decay mode of $t \rightarrow W + b$ at the tree level and the NLO can be found in Ref. [20], which we list below for the convenience of the reader,

$$\begin{aligned} \Gamma_0(t \rightarrow W + b) &= \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \beta_W^4 (3 - 2\beta_W^2), \\ \Gamma(t \rightarrow W + b) &= \Gamma_0(t \rightarrow W + b) \left\{ 1 + \frac{2\alpha_s}{3\pi} \left[2 \left(\frac{(1 - \beta_W^2)(2\beta_W^2 - 1)(\beta_W^2 - 2)}{\beta_W^4 (3 - 2\beta_W^2)} \right) \ln(1 - \beta_W^2) \right. \right. \\ &\quad \left. \left. - \frac{9 - 4\beta_W^2}{3 - 2\beta_W^2} \ln \beta_W^2 + 2\text{Li}_2(\beta_W^2) - 2\text{Li}_2(1 - \beta_W^2) - \frac{6\beta_W^4 - 3\beta_W^2 - 8}{2\beta_W^2 (3 - 2\beta_W^2)} - \pi^2 \right] \right\}, \end{aligned} \quad (14)$$

where $\beta_W \equiv \sqrt{1 - m_W^2/m_t^2}$.

IV. RENORMALIZATION GROUP EQUATION IMPROVEMENT

Due to the large scale dependence in the process $t \rightarrow q + g$, we use the renormalization group evolution to improve the result of perturbation theory. The anomalous couplings κ^g satisfy the following renormalization group equation

$$\frac{d\kappa^g}{d \ln \mu} = -\gamma_{\kappa^g} \kappa^g, \quad (15)$$

where

$$\gamma_{\kappa^g}(g_s) = -2g_s^2 \frac{dZ_{\kappa^g,1}(g_s)}{dg_s^2}. \quad (16)$$

For simplicity, we neglect the subscript tq of the anomalous couplings above and in the following discussion. $Z_{\kappa^g,1}(g_s)$ is the residue of the renormalization constant δZ_{κ^g} . Thus according to the Eq. (4), we have

$$\gamma_{\kappa^g}(g_s) = -\frac{\alpha_s}{3\pi}. \quad (17)$$

Substitute it into Eq. (15), we can solve the renormalization group equation and get

$$\kappa^g(\mu) = \kappa^g(\mu') \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu)} \right)^{\frac{2}{3\beta_0}}, \quad (18)$$

where $\beta_0 = 11 - \frac{2}{3}N_f$. For $\alpha_s(\mu)$, we take it by solving the following renormalization group equation

$$\frac{d\alpha_s(\mu)}{d \ln \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2. \quad (19)$$

We do not consider the higher order effects in the γ_{κ^g} and $\beta(\alpha_s)$ here because their effects are small numerically.

V. CONTRIBUTIONS TO $t \rightarrow q + \gamma$ AND $t \rightarrow q + Z$ FROM FCNC OPERATOR MIXING

The NLO contributions given in Eq. (11) are proportional to the LO results. However, for the decay processes $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, there are in general contributions induced from the mixing of operators. Because we have no idea about the magnitude of the coefficients κ^V , these operator mixing contributions may be significant in some cases. In this section we will present the contributions coming from the operator mixing so as to complete the full $\mathcal{O}(\alpha_s)$ corrections to the decay processes $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$.

The contributing Feynman diagrams are shown in Fig. 4 and Fig. 5. In previous section, we consider only the contributions from Fig. 4(a) and Fig. 5(a), (b). In case that all of the three κ^V are at the same order, the terms proportional to $\kappa^g \kappa^\gamma$ and $\kappa^g \kappa^Z$ could contribute to $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$ with the same significance as the ones we considered before. Thus, we will also present these contributions below to investigate the effects from operator mixing.

For convenience, we introduce the following abbreviations

$$\begin{aligned}
\Gamma_0^\gamma &= 2Q_f \alpha m_t^3 \left(\frac{\kappa_{tq}^\gamma}{\Lambda} \right) \left(\frac{\kappa_{tq}^g}{\Lambda} \right) C_\epsilon (f_{tq}^{\gamma*} f_{tq}^g + h_{tq}^{\gamma*} h_{tq}^g), \\
\Gamma_0^Z &= \frac{\alpha m_t^3 \beta_Z^{-4\epsilon}}{\sin 2\theta_W \cos \theta_W} \left(\frac{\kappa_{tq}^Z}{\Lambda} \right) \left(\frac{\kappa_{tq}^g}{\Lambda} \right) \frac{C_\epsilon}{1-\epsilon}, \\
S_3 &= \frac{s_3 - Q_f \sin^2 \theta_W}{\sin \theta_W}, \\
Q_F &= Q_f \sin \theta_W, \\
\omega_\beta &= \sqrt{(1 - \beta_Z^2)(3 + \beta_Z^2)}, \\
c_1 &= g_L^{Z*} g_L^g Q_F - g_R^{Z*} g_R^g S_3, \\
c_2 &= g_R^{Z*} g_R^g Q_F - g_L^{Z*} g_L^g S_3,
\end{aligned} \tag{20}$$

where Q_f is the electric charge quantum number of the fermions, e.g., $2/3$ for up-type quark and $-1/3$ for down-type quark. s_3 is the third component of the $SU(2)_L$. Notice that we use chiral parameters g_L^i, g_R^i instead of f_{tq}^i and h_{tq}^i in the Z -channel for simplicity, which have the following relations,

$$g_L^i = f_{tq}^i - i h_{tq}^i, \quad g_R^i = f_{tq}^i + i h_{tq}^i, \quad |g_L^i|^2 + |g_R^i|^2 = 2.$$

Denoting $\Gamma_{\text{virt,mix}}^i (i = \gamma, Z)$ as contributions to partial decay widths from the sum of Fig. 4(b)-(g) and the corresponding counter terms, we obtain

$$\begin{aligned}
\Gamma_{\text{virt,mix}}^\gamma &= \Gamma_0^\gamma \left[\frac{2\alpha_s}{3\pi} \left(-\frac{4}{\epsilon_{\text{UV}}} + 4\gamma_E - 4 \ln \frac{4\pi\mu^2}{m_t^2} - 11 + \frac{2\pi^2}{3} \right) + 2\delta Z_{g\gamma} \right], \\
\Gamma_{\text{virt,mix}}^Z &= \Gamma_0^Z \beta_Z^4 (3 - \beta_Z^2 - 2\epsilon) \left\{ \frac{\alpha_s}{3\pi} c_1 \left[-\frac{2}{\epsilon_{\text{UV}}} + 2\gamma_E - 2 \ln \frac{4\pi\mu^2}{m_t^2} - 4 + \frac{2(1 - \beta_Z^2) \ln(1 - \beta_Z^2) - 9}{3 - \beta_Z^2} \right. \right. \\
&\quad \left. - \frac{4(2 - \beta_Z^2)}{\beta_Z^2(3 - \beta_Z^2)} \sqrt{\frac{3 + \beta_Z^2}{1 - \beta_Z^2}} \tan^{-1} \left(\frac{\omega_\beta}{1 + \beta_Z^2} \right) - \frac{8\overline{C}_0}{\beta_Z^4(3 - \beta_Z^2)} \right] + \frac{\alpha_s}{3\pi} c_2 \left[-\frac{2}{\epsilon_{\text{UV}}} \right. \\
&\quad \left. \left. + 2\gamma_E - 2 \ln \frac{4\pi\mu^2}{m_t^2} + 2 \sqrt{\frac{3 + \beta_Z^2}{1 - \beta_Z^2}} \tan^{-1} \left(\frac{\omega_\beta}{1 + \beta_Z^2} \right) - \frac{9 - 4\beta_Z^2}{3 - \beta_Z^2} \right] + 2\delta Z_{gZ} \right\},
\end{aligned} \tag{21}$$

where $\overline{C_0}$ and the renormalization constants $\delta Z_{g\gamma}$ and δZ_{gZ} are given below:

$$\begin{aligned}
\overline{C_0} &= \ln \frac{(1 + \beta_Z^2)(2 - 2\beta_Z^2 - \beta_Z^4) + i\beta_Z^2(2 + \beta_Z^2)\omega_\beta}{2} \ln \frac{1 + \beta_Z^2 + i\omega_\beta}{2} \\
&\quad + 2\text{Li}_2(-\beta_Z^2) + \text{Li}_2\left(\frac{\beta_Z^2}{2}(1 + \beta_Z^2 + i\omega_\beta)\right) + \text{Li}_2\left(\frac{\beta_Z^2}{2}(1 + \beta_Z^2 - i\omega_\beta)\right) \\
&\quad - \text{Li}_2\left(\frac{\beta_Z^2}{2}(-1 + 2\beta_Z^2 + \beta_Z^4 + i(1 + \beta_Z^2)\omega_\beta)\right) \\
&\quad - \text{Li}_2\left(\frac{\beta_Z^2}{2}(-1 + 2\beta_Z^2 + \beta_Z^4 - i(1 + \beta_Z^2)\omega_\beta)\right), \\
\delta Z_{g\gamma} &= \frac{\alpha_s}{3\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \frac{4}{\epsilon_{\text{UV}}}, \\
\delta Z_{gZ} &= \frac{\alpha_s}{3\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \frac{c_1 + c_2}{\epsilon_{\text{UV}}}. \tag{22}
\end{aligned}$$

We can see that $\Gamma_{\text{virt,mix}}^\gamma$ and $\delta Z_{g\gamma}$ are consistent with Eqs. (4.8) and (4.10) in Ref. [21], up to a factor of $\frac{1}{3}$, which is absorbed into Q_f in our formula. Besides, $\Gamma_{\text{virt,mix}}^Z$ and δZ_{gZ} are consistent with $\Gamma_{\text{virt,mix}}^\gamma$ and $\delta Z_{g\gamma}$ if we take the limit $M_Z \rightarrow 0$ and $Q_F = -S_3 = 1$.

It worths to briefly discuss the renormalization constant δZ_{gZ} here. To renormalize the operator involving the Z boson, we need to introduce two renormalization constants: one for vector current and another for axial vector current (or left-hand and right-hand). Due to the similarity between the structure of the anomalous couplings of γ and Z bosons in Eq. (1), we expect that $\Gamma_{\text{virt,mix}}^\gamma$ and $\Gamma_{\text{virt,mix}}^Z$ are equal in the limit of $M_Z \rightarrow 0$ and $Q_F = -S_3 = 1$. Thus, we introduce the renormalization constants at the level of the decay width in contrast to the usual practice which is done at the level of the scattering amplitude. Nevertheless, this treatment would simplify our calculation, so we take it as the definition of δZ_{gZ} in this paper.

Denoting $\mathcal{M}_A^i(i = \gamma, Z)$ as the sum of Fig. 5 (a) and (b), and $\mathcal{M}_B^i(i = \gamma, Z)$ as the sum

of Fig. 5 (c) and (d), we can present the contributions $\Gamma_{\text{real,mix}}^i$ as

$$\begin{aligned}
\Gamma_{\text{real,mix}}^\gamma &= \frac{1}{m_t} \int \sum |\overline{\mathcal{M}_A^{\gamma*} \mathcal{M}_B^\gamma}| d\Phi = \Gamma_0^\gamma \frac{2\alpha_s}{9\pi} (2\pi^2 - 25), \\
\Gamma_{\text{real,mix}}^Z &= \frac{1}{m_t} \int \sum |\overline{\mathcal{M}_A^{Z*} \mathcal{M}_B^Z}| d\Phi \\
&= \Gamma_0^Z \frac{\alpha_s}{3\pi} \left\{ \frac{\beta_Z^2}{3} [(-\beta_Z^4 + 21\beta_Z^2 - 72)c_1 + (24 - 21\beta_Z^2 - \beta_Z^4)c_2] \right. \\
&\quad + 2\omega_\beta [-(1 + 3\beta_Z^2)c_1 + (1 - \beta_Z^2)c_2] \left[\tan^{-1} \left(\frac{-1 - \beta_Z^2}{\omega_\beta} \right) + \tan^{-1} \left(\frac{1 - \beta_Z^2}{\omega_\beta} \right) \right] \\
&\quad + (1 - \beta_Z^2) \ln(1 - \beta_Z^2) [(9\beta_Z^2 - 21)c_1 + (5 - \beta_Z^2)c_2] + 8c_1 \\
&\quad \times \left[-\text{Li}_2(\beta_Z^2) + \text{Li}_2 \left(\frac{\beta_Z^2}{2} (1 + \beta_Z^2 + i\omega_\beta) \right) + \text{Li}_2 \left(\frac{\beta_Z^2}{2} (1 + \beta_Z^2 - i\omega_\beta) \right) \right] \left. \right\} \quad (23)
\end{aligned}$$

Combine these results together, we have

$$\begin{aligned}
\Gamma_{\text{mix}}^\gamma &= \Gamma_{\text{virt,mix}}^\gamma + \Gamma_{\text{real,mix}}^\gamma = \Gamma_0^\gamma \frac{4\alpha_s}{9\pi} \left(6 \ln \frac{m_t^2}{\mu^2} + 2\pi^2 - 29 \right), \\
\Gamma_{\text{mix}}^Z &= \Gamma_{\text{virt,mix}}^Z + \Gamma_{\text{real,mix}}^Z \\
&= \frac{\alpha_s}{3\pi} \Gamma_0^Z \left\{ \frac{\beta_Z^2}{3} [c_1(-72 - 42\beta_Z^2 + 11\beta_Z^4) + c_2(24 - 48\beta_Z^2 + 11\beta_Z^4)] \right. \\
&\quad + (1 - \beta_Z^2) [(2\beta_Z^4 + 9\beta_Z^2 - 21)c_1 + (5 - \beta_Z^2)c_2] \ln(1 - \beta_Z^2) \\
&\quad + 2\beta_Z^2 \frac{\omega_\beta}{1 - \beta_Z^2} \tan^{-1} \left(\frac{\omega_\beta}{1 + \beta_Z^2} \right) [\beta_Z^2(3 - \beta_Z^2)c_2 - 2(2 - \beta_Z^2)c_1] \\
&\quad + 2\omega_\beta [-(1 + 3\beta_Z^2)c_1 + (1 - \beta_Z^2)c_2] \left[\tan^{-1} \left(\frac{-1 - \beta_Z^2}{\omega_\beta} \right) + \tan^{-1} \left(\frac{1 - \beta_Z^2}{\omega_\beta} \right) \right] \\
&\quad \left. + 2\beta_Z^4(3 - \beta_Z^2)(c_1 + c_2) \ln \frac{m_t^2}{\mu^2} + 8c_1 X \right\}, \quad (24)
\end{aligned}$$

where

$$\begin{aligned}
X &= \text{Li}_2 \left(\frac{\beta_Z^2}{2} (-1 + 2\beta_Z^2 + \beta_Z^4 + i(1 + \beta_Z^2)\omega_\beta) \right) + \text{Li}_2 \left(\frac{\beta_Z^2}{2} (-1 + 2\beta_Z^2 + \beta_Z^4 - i(1 + \beta_Z^2)\omega_\beta) \right) \\
&\quad - \text{Li}_2(\beta_Z^2) - 2\text{Li}_2(-\beta_Z^2) - \ln \frac{1 + \beta_Z^2 + i\omega_\beta}{2} \ln \frac{(1 + \beta_Z^2)(2 - 2\beta_Z^2 - \beta_Z^4) + i\beta_Z^2(2 + \beta_Z^2)\omega_\beta}{2}. \quad (25)
\end{aligned}$$

VI. NUMERICAL ANALYSIS TO TOP FCNC DECAY

For the numerical calculation of the branching ratios, we take the SM parameters as given in Ref. [22]:

$$m_t = 171.2 \text{ GeV}, \quad N_f = 5, \quad m_W = 80.398 \text{ GeV},$$

TABLE I: Numerical results of branching ratios. Here $\mu = m_t$, $\kappa_{tq}^V/\Lambda = 1\text{TeV}^{-1}$, $f_{tq}^{\gamma*}f_{tq}^g + h_{tq}^{\gamma*}h_{tq}^g = 1$ and $g_R^{Z*}g_R^g = g_L^{Z*}g_L^g = 1$.

| BR | BR _{LO} | BR _{NLO} | BR _{tot} | BR _{NLO} /BR _{LO} | BR _{tot} /BR _{LO} |
|----------------------------|------------------|-------------------|-------------------|-------------------------------------|-------------------------------------|
| $t \rightarrow q + g$ | 1.0010 | 1.1964 | - | 1.195 | - |
| $t \rightarrow q + \gamma$ | 0.0544 | 0.0542 | 0.0486 | 0.996 | 0.893 |
| $t \rightarrow q + Z$ | 0.04484 | 0.04480 | 0.0459 | 0.999 | 1.025 |

TABLE II: Numerical results of the partial decay width. Here $\mu = m_t$, $\kappa_{tq}^V/\Lambda = 1\text{TeV}^{-1}$, $f_{tq}^{\gamma*}f_{tq}^g + h_{tq}^{\gamma*}h_{tq}^g = 1$ and $g_R^{Z*}g_R^g = g_L^{Z*}g_L^g = 1$.

| Width[in unit GeV] | Γ_{LO} | Γ | Γ_{tot} | $\Gamma/\Gamma_{\text{LO}}$ | $\Gamma_{\text{tot}}/\Gamma_{\text{LO}}$ |
|----------------------------|----------------------|----------|-----------------------|-----------------------------|--|
| $t \rightarrow q + g$ | 1.443 | 1.577 | - | 1.09 | - |
| $t \rightarrow q + \gamma$ | 0.078 | 0.071 | 0.064 | 0.91 | 0.82 |
| $t \rightarrow q + Z$ | 0.065 | 0.059 | 0.061 | 0.91 | 0.94 |

$$m_Z = 91.1876\text{GeV}, \quad \alpha = 1/128, \quad \sin^2 \theta_W = 0.231,$$

$$V_{tb} = 1, \quad G_F = 1.16637 \times 10^{-5}\text{GeV}^{-2}.$$

We analyze our results by choosing a special set of parameters and fix $\mu = m_t$ in the following analysis unless specified. Table I and Table II show the $\mathcal{O}(\alpha_s)$ effects to various decay branching ratios and partial decay widths, respectively, where $\Gamma_{\text{tot}} = \Gamma + \Gamma_{\text{mix}}$ and $\text{BR}_{\text{tot}} = \Gamma_{\text{tot}}/\Gamma(t \rightarrow W + b)$. From Table I, we see that the NLO correction increases the LO branching ratio by about 20% for $t \rightarrow q + g$, while the NLO corrections are much smaller for the other two decay modes. But after including the operator mixing effects, the branching ratio for $t \rightarrow q + \gamma$ can decrease by about 10%, assuming $\frac{\kappa_{tq}^g}{\Lambda} = \frac{\kappa_{tq}^\gamma}{\Lambda} = 1\text{TeV}^{-1}$ and $f_{tq}^{\gamma*}f_{tq}^g + h_{tq}^{\gamma*}h_{tq}^g = 1$. For $t \rightarrow q + Z$, the branching ratio can increase by about 2%, assuming $\frac{\kappa_{tq}^g}{\Lambda} = \frac{\kappa_{tq}^Z}{\Lambda} = 1\text{TeV}^{-1}$ and $g_R^{Z*}g_R^g = g_L^{Z*}g_L^g = 1$. From Table II we can see that for the partial decay width the NLO results modify the LO results by about 9% in magnitude for all the three modes, and the operator mixing effects can decrease the partial width by about 9% and increase by about 3% with the above assumptions of parameters for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, respectively.

For convenience, we show the branching ratio of $t \rightarrow q + g$ as a function of κ^g/Λ in Fig. 6.

Using the upper limits measured by the D0 Collaboration at the Tevatron [1], we get the following results:

$$\begin{aligned}\frac{\kappa_{tc}^g}{\Lambda} < 0.15 \text{TeV}^{-1} &\Rightarrow \text{BR}(t \rightarrow c + g) < 2.69 \times 10^{-2}, \\ \frac{\kappa_{tu}^g}{\Lambda} < 0.037 \text{TeV}^{-1} &\Rightarrow \text{BR}(t \rightarrow u + g) < 1.64 \times 10^{-3}.\end{aligned}\quad (26)$$

Using our previous results [3, 4, 19], CDF Collaboration presented a more precise results for the anomalous couplings and the branching ratios in a recent letter [2]

$$\begin{aligned}\frac{\kappa_{tc}^g}{\Lambda} < 0.069 \text{TeV}^{-1} &\Rightarrow \text{BR}(t \rightarrow c + g) < 5.7 \times 10^{-3}, \\ \frac{\kappa_{tu}^g}{\Lambda} < 0.018 \text{TeV}^{-1} &\Rightarrow \text{BR}(t \rightarrow u + g) < 3.9 \times 10^{-4}.\end{aligned}\quad (27)$$

Following the analysis in Ref. [5], we plot the coupling κ_{tq}^g/Λ as a function of the branching ratio in Fig. 7, where the ATLAS sensitivities for the two different expected integrated luminosities are also exhibited. From Fig. 7, we can see that the NLO prediction improves the sensitivities of the LHC experiments to measuring the top quark FCNC couplings. With an integrated luminosity of 10fb^{-1} , the ATLAS experiment sensitivities can be translated to the following relations on FCNC couplings:

$$\begin{aligned}\text{BR}(t \rightarrow q + g) > 1.3 \times 10^{-3} &\Rightarrow \frac{\kappa_{tq}^g}{\Lambda} > 0.033 \text{TeV}^{-1}, \\ \text{BR}(t \rightarrow q + \gamma) > 4.1 \times 10^{-5} &\Rightarrow \frac{\kappa_{tq}^\gamma}{\Lambda} > 0.028 \text{TeV}^{-1}, \\ \text{BR}(t \rightarrow q + Z) > 3.1 \times 10^{-4} &\Rightarrow \frac{\kappa_{tq}^Z}{\Lambda} > 0.083 \text{TeV}^{-1},\end{aligned}\quad (28)$$

and with an integrated luminosity of 100fb^{-1} , they can be translated to the more stringent relations:

$$\begin{aligned}\text{BR}(t \rightarrow q + g) > 4.2 \times 10^{-4} &\Rightarrow \frac{\kappa_{tq}^g}{\Lambda} > 0.019 \text{TeV}^{-1}, \\ \text{BR}(t \rightarrow q + \gamma) > 1.2 \times 10^{-5} &\Rightarrow \frac{\kappa_{tq}^\gamma}{\Lambda} > 0.015 \text{TeV}^{-1}, \\ \text{BR}(t \rightarrow q + Z) > 6.1 \times 10^{-5} &\Rightarrow \frac{\kappa_{tq}^Z}{\Lambda} > 0.037 \text{TeV}^{-1}.\end{aligned}\quad (29)$$

Finally, we illustrate the fact that the NLO prediction reduces the theoretical uncertainty in its prediction on the decay branching ratios and partial decay widths of the top quark. We define $R_{\text{LO}}(\mu) = \Gamma_0(\mu)/\Gamma_0(\mu = m_t)$ and $R_{\text{NLO}}(\mu) = \Gamma(\mu)/\Gamma(\mu = m_t)$, and show the value

of $R(\mu)$ as a function of μ for $t \rightarrow q + g$ in Fig. 8. It shows that the theoretical uncertainty from the renormalization scale dependence can be largely reduced to a couple of percent once the NLO calculation is taken into account, so the NLO results give much more reliable theoretical predictions.

To investigate the contributions from the operator mixing effects, we present the contour curves for the variables $\text{Re}(f^{\gamma*}f^g)$ (or $\text{Re}(g_L^{Z*}g_L)$) and $\text{Re}(h^{\gamma*}h^g)$ (or $\text{Re}(g_R^{Z*}g_R)$) for the γ (or Z) channel, where $\frac{\kappa^\gamma}{\Lambda} = \frac{\kappa^g}{\Lambda} = 1 \text{ TeV}^{-1}$ (or $\frac{\kappa^Z}{\Lambda} = \frac{\kappa^g}{\Lambda} = 1 \text{ TeV}^{-1}$). In Fig. 9 (a), we show the pure operator mixing effects to the branching ratio of $t \rightarrow q + \gamma$, while in Fig. 9 (b) the total result are shown, which includes the LO, NLO and mixing effects all together. Similarly, we present the results for $t \rightarrow q + Z$ in Fig. 10.

Considering the mixing effects, we need to modify the inequalities involving γ and Z in Eq. (28) and Eq. (29), which read as

$$\begin{aligned}
&\text{BR}(t \rightarrow q + \gamma) > 4.1 \times 10^{-5} \Rightarrow \\
&\left(\frac{\kappa_{tq}^\gamma}{\Lambda}\right)^2 - 0.1 \left(\frac{\kappa_{tq}^\gamma}{\Lambda}\right) \left(\frac{\kappa_{tq}^g}{\Lambda}\right) \text{Re}(f_{tq}^{\gamma*}f_{tq}^g + h_{tq}^{\gamma*}h_{tq}^g) > 7.6 \times 10^{-4} \text{TeV}^{-2}, \\
&\text{BR}(t \rightarrow q + Z) > 3.1 \times 10^{-4} \Rightarrow \\
&\left(\frac{\kappa_{tq}^Z}{\Lambda}\right)^2 + [-3.6\text{Re}(g_L^{Z*}g_L^g) + 6.2\text{Re}(g_R^{Z*}g_R^g)] \left(\frac{\kappa_{tq}^Z}{\Lambda}\right) \left(\frac{\kappa_{tq}^g}{\Lambda}\right) \times 10^{-2} > 6.9 \times 10^{-3} \text{TeV}^{-2}
\end{aligned} \tag{30}$$

for the integrated luminosity of 10fb^{-1} , and

$$\begin{aligned}
&\text{BR}(t \rightarrow q + \gamma) > 1.2 \times 10^{-5} \Rightarrow \\
&\left(\frac{\kappa_{tq}^\gamma}{\Lambda}\right)^2 - 0.1 \left(\frac{\kappa_{tq}^\gamma}{\Lambda}\right) \left(\frac{\kappa_{tq}^g}{\Lambda}\right) \text{Re}(f_{tq}^{\gamma*}f_{tq}^g + h_{tq}^{\gamma*}h_{tq}^g) > 2.2 \times 10^{-4} \text{TeV}^{-2}, \\
&\text{BR}(t \rightarrow q + Z) > 6.1 \times 10^{-5} \Rightarrow \\
&\left(\frac{\kappa_{tq}^Z}{\Lambda}\right)^2 + [-3.6\text{Re}(g_L^{Z*}g_L^g) + 6.2\text{Re}(g_R^{Z*}g_R^g)] \left(\frac{\kappa_{tq}^Z}{\Lambda}\right) \left(\frac{\kappa_{tq}^g}{\Lambda}\right) \times 10^{-2} > 1.4 \times 10^{-3} \text{TeV}^{-2}
\end{aligned} \tag{31}$$

for the integrated luminosity of 100fb^{-1} , respectively.

VII. CONCLUSIONS

In order to perform consistent studies for both the top quark production and decay via FCNC couplings, we have calculated the NLO QCD corrections to the three decay modes of the top quark induced by model-independent FCNC couplings of dimension-five operators. For $t \rightarrow q + g$, the NLO results increase the experimental sensitivity to the anomalous couplings. Our results show that the NLO QCD corrections enhance the LO branching ratio by about 20%, as presented in Ref. [19]. Moreover, the NLO QCD corrections vastly reduce the dependence on the renormalization scale, which leads to increased confidence in our theoretical predictions based on these results. For $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, the NLO corrections are minuscule in branching ratios, albeit they can decrease the LO widths by about 9%. However, if we further consider the effects induced from operator mixing, they can either be large or small, and increase or decrease the branching ratios for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$, depending on the values of the anomalous couplings ($\kappa_{tq}^{g,\gamma,Z}/\Lambda$, $f_{tq}^{g,\gamma,Z}$ and $h_{tq}^{g,\gamma,Z}$).

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Note added. After we completed this work we became aware of a paper by Jure Drobnak, Svjetlana Fajfer and Jernej F. Kamenik (arXiv:1004.0620) [23], where they consider the same effects from FCNC operator mixing.

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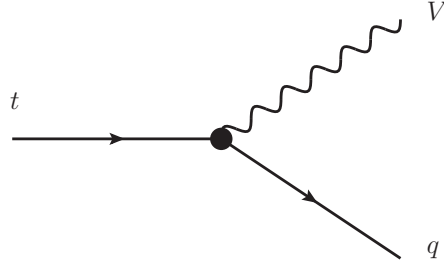


FIG. 1: Tree-level Feynman diagram for $t \rightarrow q + V$. Here, we use q to represents the up-quark and the charm-quark.

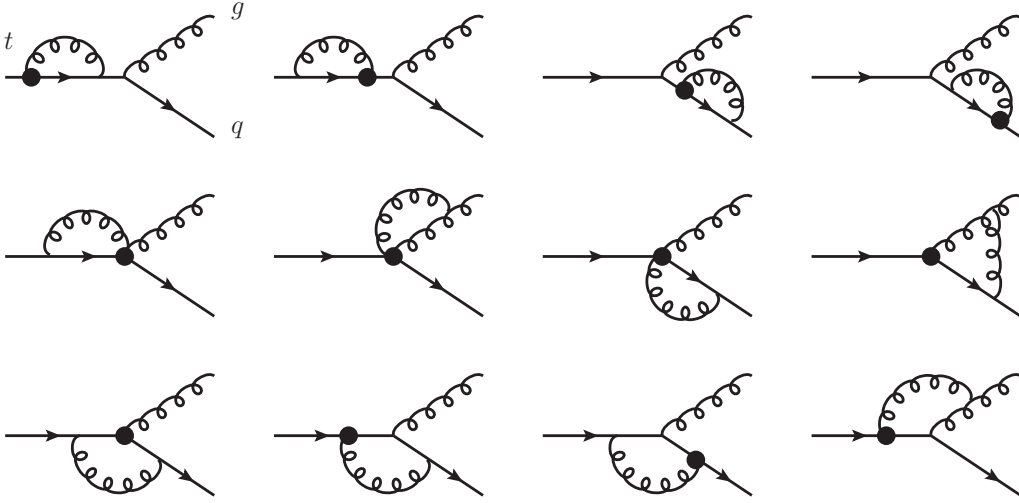


FIG. 2: One-loop Feynman diagrams for $t \rightarrow q + g$. Here we use q to represents the up-quark and the charm-quark.

[23] J. Drobnak, S. Fajfer and J. F. Kamenik, arXiv:1004.0620 [hep-ph].

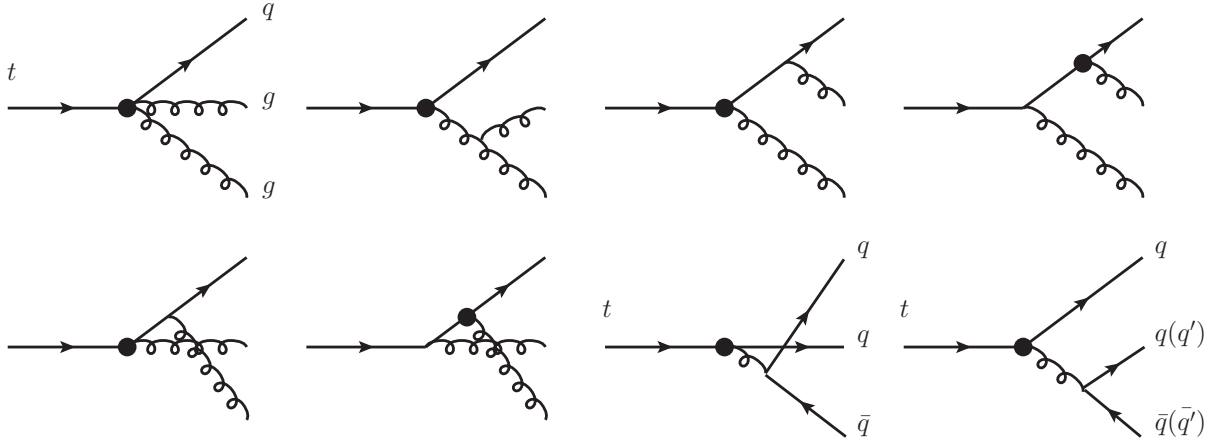


FIG. 3: Feynman diagrams of real gluon emission and gluon split. Here we use q to represents the up-quark and the charm-quark.

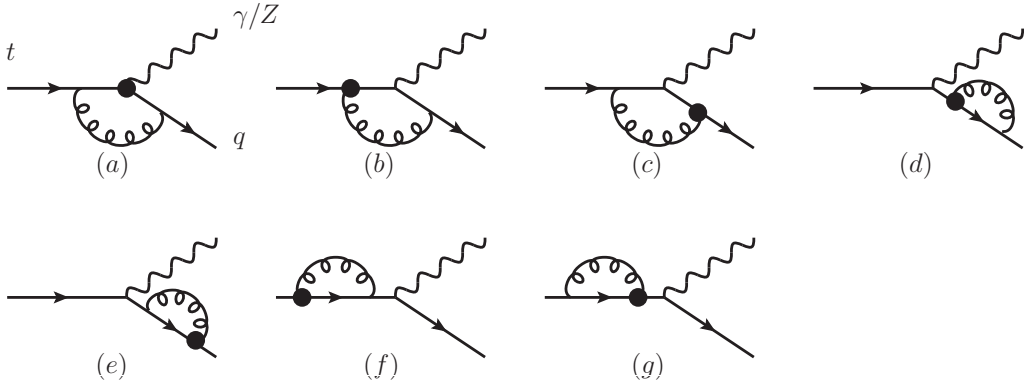


FIG. 4: Feynman diagrams of virtual corrections for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$. Here we use q to represent the up-quark and charm-quark.

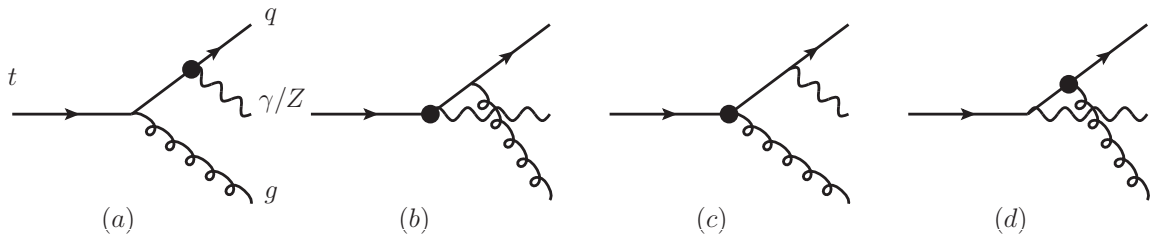


FIG. 5: Feynman diagrams of real gluon emission for $t \rightarrow q + \gamma$ and $t \rightarrow q + Z$. Here we use q to represent the up-quark and charm-quark.

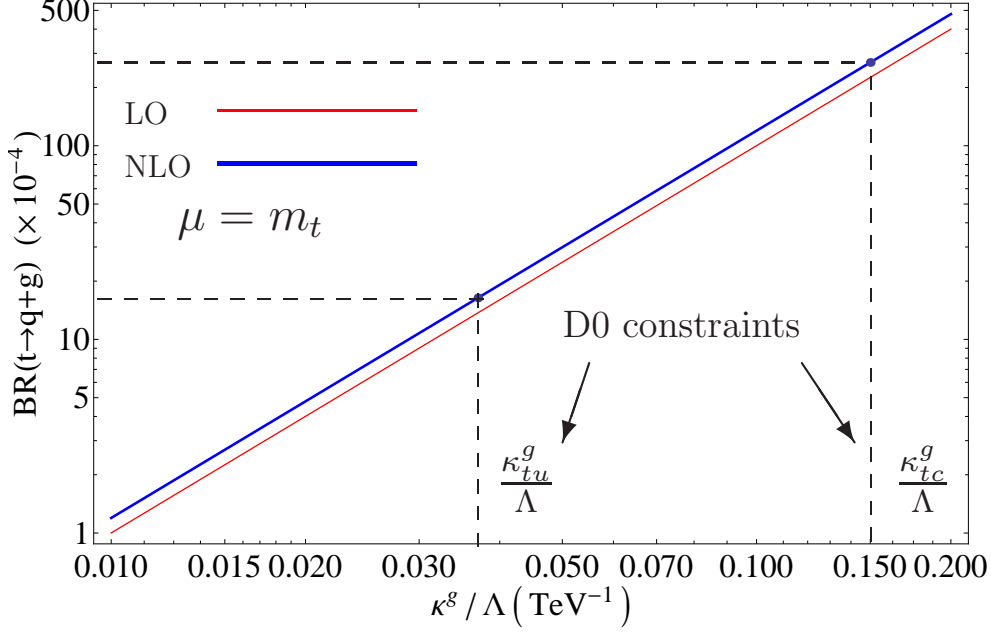


FIG. 6: Branching ratio as a function of $\frac{\kappa^g}{\Lambda}$. Here $\mu = m_t$. We also give the D0 limits from Ref. [1].

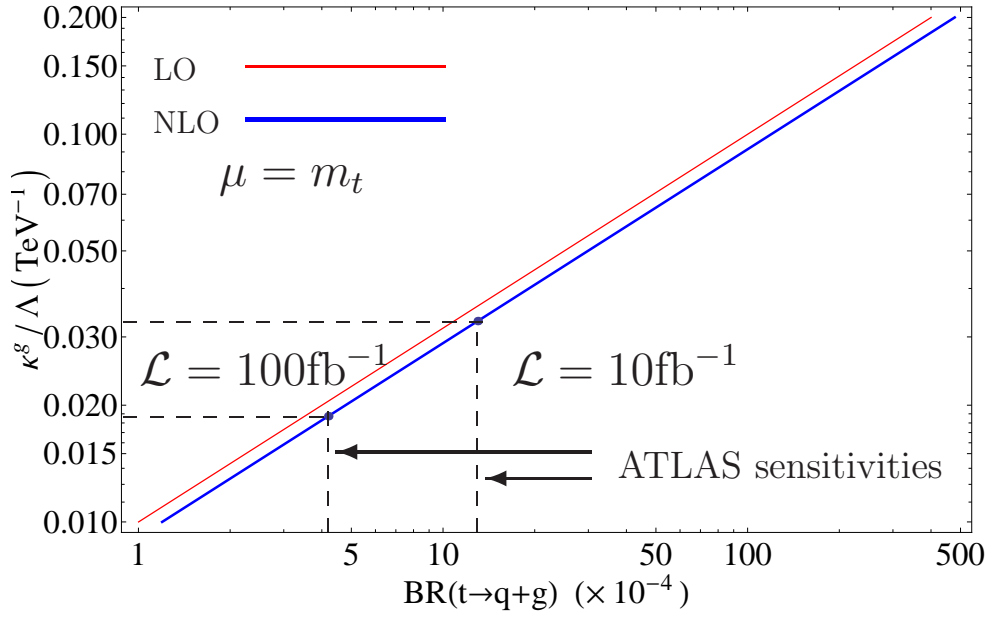


FIG. 7: $\frac{\kappa^g_{tq}}{\Lambda}$ as a function of branching ratio. Here $\mu = m_t$. We also show the ATLAS sensitivities from Ref. [5].

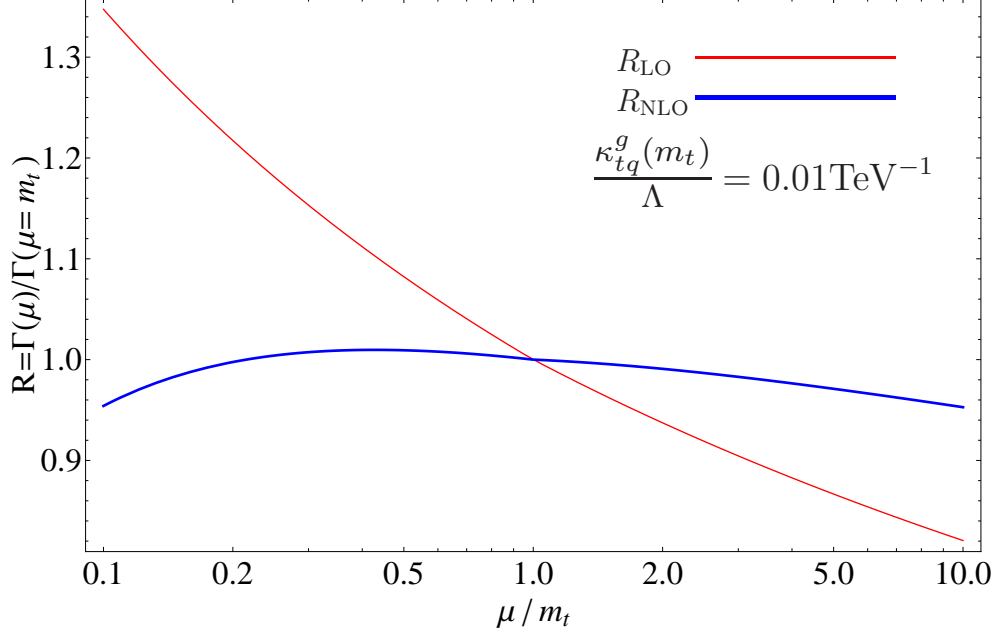


FIG. 8: The ratio R as a function of the renormalization scale. Here, $\frac{\kappa^g}{\Lambda} = 0.01 \text{ TeV}^{-1}$.

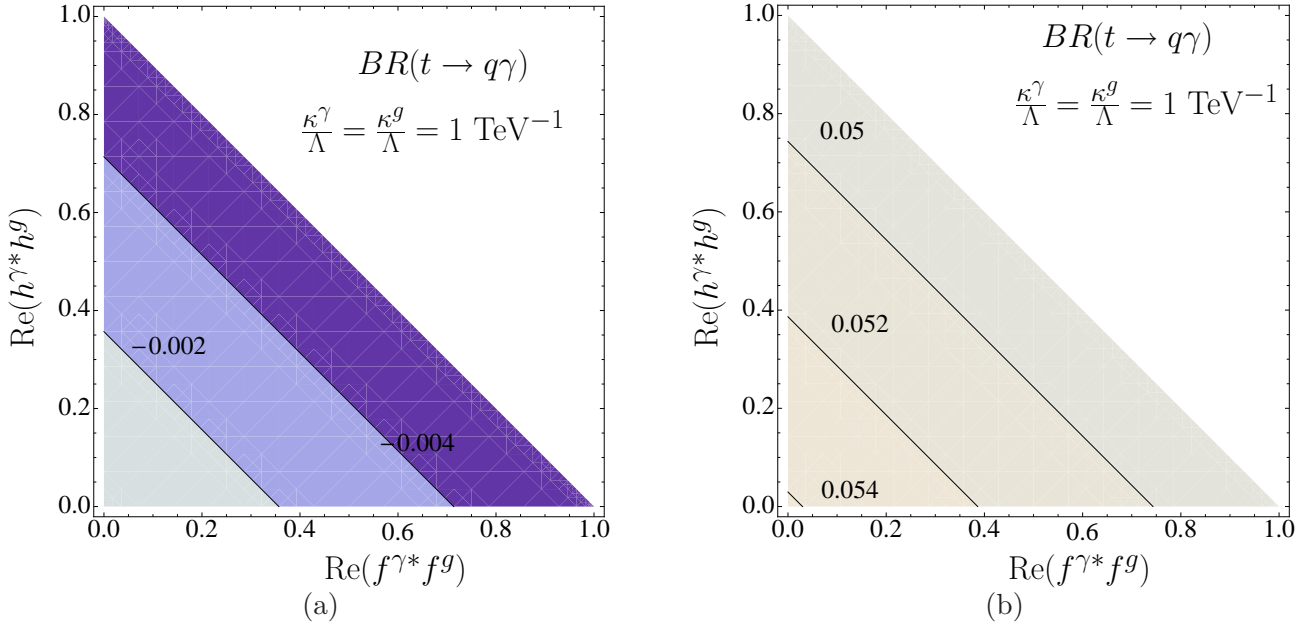


FIG. 9: The contour curves of the decay branching ratio $BR(t \rightarrow q + \gamma)$ versus the variables $Re(f^{\gamma*}f^g)$ and $Re(h^{\gamma*}h^g)$. We show in (a) contributions induced from operator mixing effects; (b) contributions including LO, NLO and mixing effects all together. Here, we set $\frac{\kappa^\gamma}{\Lambda} = \frac{\kappa^g}{\Lambda} = 1 \text{ TeV}^{-1}$ for simplicity.

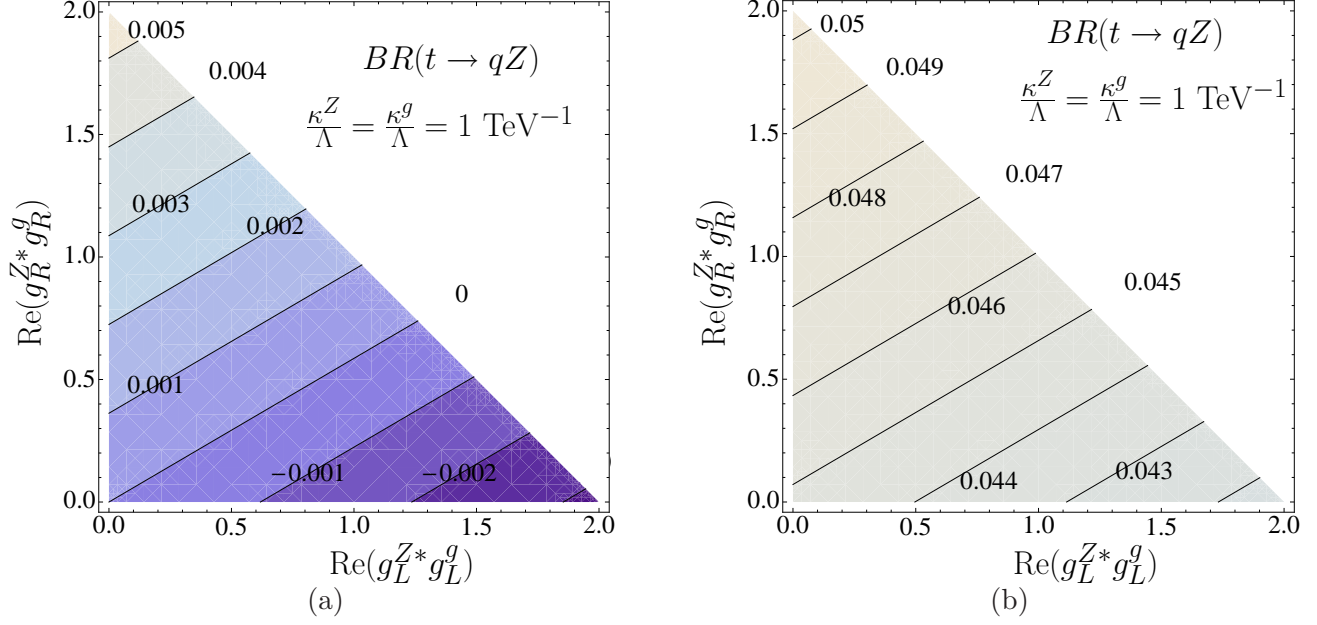


FIG. 10: The contour curves of the decay branching ratio $BR(t \rightarrow q + Z)$ versus the variables $Re(g_L^{Z*} g_L^g)$ and $Re(g_R^{Z*} g_R^g)$. We show in (a) contributions induced from operator mixing effects; (b) contributions including LO, NLO and mixing effects. Here, we set $\frac{\kappa^Z}{\Lambda} = \frac{\kappa^g}{\Lambda} = 1 \text{ TeV}^{-1}$ for simplicity.